

Abstract

Let $U_F(x)$ be the number of integers not exceeding x that can be represented by a primitive positive definite binary quadratic form $F \in \mathbb{Z}[x, y]$ having discriminant $D < 0$. It is shown that

$$U_F(x) \gg_{\varepsilon} |D|^{-\varepsilon} x (\log x)^{-\frac{1}{2}}$$

uniformly in $|D| \leq (\log x)^{\log 2 - \varepsilon}$ and

$$U_F(x) \gg_{\varepsilon} x (\log x)^{-1 - \kappa(\log(2\kappa) - 1) - \varepsilon}$$

uniformly in $|D| \leq (\log x)^{2\kappa \log 2 - \varepsilon}$ for any $\frac{1}{2} \leq \kappa \leq \frac{1}{1 + \log 2} - \varepsilon$.

As an application a problem of Erdős is considered. Let $V(x)$ be the number of integers representable as a sum of two squareful integers. Then $V(x) \gg x (\log x)^{-0.253}$.

MSC (2000) *11N25, 11E16, 11E25, 11N37