Minisymposium 7 - Stochastic algorithms and Markov processes

Solving the filtering problem in a continuous time framework. Advantages and Pitfalls

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Particle filters have enjoyed a period of fast development in the last fifteen years both from the theoretical and from the applicative viewpoint. For many filtering problems, a natural mathematical model for the signal is a continuous time Markov process that satisfies a stochastic diferential equation of the form

$$dx_t = f(x_t)\,dt + \sigma(x_t)\,dv_t, \tag{1}$$

where v is a Wiener process whilst the observation is modelled by an evolution equation of the form

$$dy_t = h(x_t)\,dt + dw_t.$$

where w is a Wiener process independent of v.
Within the continuous time framework, $\pi = \{\pi_t, t \geq 0\}$ the conditional distribution of the signal x_t given the observation data $\{y_s, s \in [0,T]\}$ is the solution of a nonlinear stochastic PDE, called the Kushner-Stratonovitch with no explicit solution in the general case. For a suitable class of functions $\varphi,\ \pi_t(\varphi)$ can be viewed as the expected value of a certain functional *parametrized* by the observation path $\{y_s, s \in [0,T]\}$ of a process ξ which is a solution of (1). In other words, we seek to obtain something akin to what in the theory of approximation for stochastic differential equations is called a *weak solution* of (1).
This fundamental observation leads to approximating algorithms for the filtering problem obtained by adapting existing weak approximations of SDEs to the filtering framework. Firstly, one approximates π by replacing the (continuous) observation path with a discrete version. The standard method is to choose an equidistant partition $\{i\delta,\ i = 0, 1, ...\}$ of the timeline and consider only the observation data $\{y_{i\delta},\ i = 0, 1, ...\}$ corresponding to the partition time instances. The resulting probability measure π^δ converges to π as δ tends to 0. We present a number of convergence results regarding for approximations of π^δ.