Minisymposium 19

Random Discrete Structures and Algorithms

Leiter des Symposiums:

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Donnerstag, 21. September

Übungsraum 2, Geographisches Institut, Meckenheimer Allee 166

15:00 – 15:25 **Ralph Neininger** *(Frankfurt)*
Probabilistic analysis of game tree evaluation

15:30 – 15:55 **Christian Scheideler** *(TU München)*
Adversarial mixing in virtual space

16:00 – 16:25 **Benjamin Doerr** *(MPI Saarbrücken)*
Dependant Randomized Rounding

16:30 – 16:55 **Bernd Gärtner** *(ETH Zürich)*
Clarkson's randomized linear programming algorithms revisited

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17:00 – 17:25 **Anand Srivastav** *(Kiel)*
Euclidean Nearest Neighbor Problems for Random Points

17:30 – 17:55 **Berthold Vöcking** *(Aachen)*
Probabilistic Analysis of Local Search Algorithms for TSP

Freitag, 22. September

Übungsraum 2, Geographisches Institut, Meckenheimer Allee 166

15:00 – 15:25 **Volker Kaibel** *(TU Berlin, ZIB)*
0/1-Polytopes With Exponentially Small Vertex-Expansion

15:30 – 15:55 **Mihyun Kang** *(HU Berlin)*
Zufällige planare Strukturen

16:00 – 16:25 **Amin Coja-Oghlan** *(HU Berlin)*
Central and local limit theorems for the giant component

16:30 – 16:55 **Dieter Rautenbach** *(Bonn)*
Beyond Acyclic Colorings

17:00 – 17:25 **Mathias Schacht** *(HU Berlin)*
On the bandwidth conjecture of Bollobás and Komlós

Vortragsauszüge

Ralph Neininger *(Frankfurt)*
Probabilistic analysis of game tree evaluation

In the analysis of game-searching methods a basic problem is to determine the value of the root of a minimax tree with certain given numbers at its leaves (the input). Various models for the input and related evaluation algorithms have been proposed and analyzed.
We review some of these models, in particular models with input from the set $\{0, 1\}$ and probabilistic models such as Pearl's model and the (random) incremental model. We discuss the complexity of evaluation algorithms under these models: a new tail bound for the complexity of Snir's randomized evaluation algorithm is given improving upon a Gaussian tail bound due to Karp and Zhang. Also a limit law of the root's value in Pearl's model is given leading to a conjecture on the asymptotic distribution of the complexity of α-β pruning.

The talk is based on the papers

Ali Khan, T. and R. Neininger (2004) Probabilistic analysis for randomized game tree evaluation. *Mathematics and Computer Science III (Vienna 2004)*, 163-174, Trends in Mathematics, Birkhuser, Basel.

Ali Khan, T., Devroye, L. and R. Neininger (2005) A limit law for the root value of minimax trees. *Electronic Communications in Probability* **10**, 273-281.

Christian Scheideler *(TU München)*
Adversarial mixing in virtual space

In this talk I will study the problem of how to keep honest and adversarial nodes well-spread in a virtual space. More precisely, I will be focusing on the space [0,1) of real numbers. At any time, every node in the system is assigned to some point in that space, and nodes may join or leave the system as they like. The problem is to find simple and efficient join and leave protocols for the nodes that are oblivious to their state yet can preserve the property that, given n nodes in the system, the following conditions are met: For any interval I of size $(c \log n)/n$, where c is a sufficiently large constant,

- I contains $\Theta(|I| \cdot n)$ nodes, and
- the honest nodes in I are in the majority.

First, I consider the case that the adversary can only issue adaptive join/leave requests for adversarial nodes and then the case that the adversary can issue adaptive join/leave requests for all nodes. For both cases, simple join and leave protocols can be constructed that preserve the conditions above, with high probability. This has interesting applications for peer-to-peer systems.

Benjamin Doerr *(MPI Saarbrücken)*
Dependant Randomized Rounding

A very successful approach to rounding problems is the one of *randomized rounding* introduced by Raghavan and Thompson (1987). Here, if $x \in [0,1]^n$, its (independent) randomized rounding y satisfies $\Pr(y_j = 1) = x_j$ and $\Pr(y_j = 0) = 1 - x_j$ independently for all $j \in [n]$. Rounding independently allows to use large deviation bounds for sums of independent random variables, which makes this a powerful method in algorithmics.
A recent development in this field are dependent approaches. Of particular interest are randomized roundings that satisfy certain cardinality constraints (sums of some variables remain unchanged). In addition to satisfying such constraints we still want the roundings to be independent enough to admit large deviation bounds.
In the talk, I will present a fairly general approach to such problems. It provides simpler and faster solutions to such rounding problems regarded in the past and has new applications that could not be solved with the earlier approaches.

Bernd Gärtner *(ETH Zürich)*
Clarkson's randomized linear programming algorithms revisited

Unique sink orientations and *violator spaces* are abstract problem classes that cover linear programming but also more general problems on which simplex-type methods may cycle even in nondegenerate situations. In this talk, I will discuss the behavior of Clarkson's randomized linear programming algorithms on problems from these classes. The goal is to extract the combinatorial properties that make the algorithms work (efficiently).

Anand Srivastav *(Kiel)*
Euclidean Nearest Neighbor Problems for Random Points

In this talk we give a probabilistic analysis for the all nearest neighbor problem for two point sets uniformly distributed in the d-dimensional unit cube. While the computation of the total nearest neighbor graph can be done with basic combinatorial arguments, the proof of concentration results seems to depend on the dimension.

The paper is joint work with Andreas Baltz and Soeren Werth, Institut für Informatik, Universität zu Kiel.

Berthold Vöcking *(Aachen)*
Probabilistic Analysis of Local Search Algorithms for TSP

2-Opt is probably the most basic and widely used local search heuristic for TSP. This heuristic achieves amazingly good results on “real world” instances both with respect to running time and approximation ratio. We present a probabilistic analysis showing that the expected number of improvement steps until 2-Opt terminates on Euclidean instances in which n points are placed uniformly at random in the plane is $\tilde{O}(n^{3+5/6})$ when starting with an initial tour computed by any greedy insertion heuristic. The best previous bound was $\tilde{O}(n^{10})$.

Our probabilistic analysis is not restricted to uniformly random instances. In principle, points can be placed by arbitrary independent continuous distributions with finite support and bounded density. In particular, different points can have different distributions. Our results can be expressed in terms of a *smoothed analysis* in which an adversary selects the initial set of points from $[0, 1]^2$ and then these points are randomly perturbed with a Gaussian distribution with standard deviation σ. In this model, we obtain an upper bound of $\tilde{O}(n^{3+5/6}/\sigma)$ on the running time of 2-Opt.

Furthermore, we investigate the behavior of 2-Opt on other input models, e.g., randomly perturbed graphs, and we present an analysis of the approximation achieved by 2-Opt.

Joint work with Matthias Englert and Heiko Roeglin, RWTH Aachen.

Volker Kaibel *(TU Berlin, ZIB)*
0/1-Polytopes With Exponentially Small Vertex-Expansion

A long-standing conjecture by Mihail and Vazirani states that the graphs of 0/1-polytopes have edge-expansion at least one. A proof of this conjecture would have many important implications in the theory of randomized approximate counting. By a probabilistic construction, we show that there are d-dimensional 0/1-polytopes whose graphs have exponentially small (in d) vertex-expansion. While this may be seen as an indication that the Mihail-Vazirani conjecture is not true, we also show that our approach does not lead to a counterexample to this conjecture.

Mihyun Kang *(HU Berlin)*
Zufällige planare Strukturen

In letzter Zeit haben zufällige planare Strukturen, wie planare Graphen und outerplanare Graphen, viel Aufmerksamkeit auf sich gezogen. Typischerweise stellt man die folgenden Fragen:

- Wieviele planare Strukturen gibt es?
- Kann man eine zufällige planare Struktur gleichverteilt generieren?
- Welche Eigenschaften hat eine zufällige planare Struktur mit hoher Wahrscheinlichkeit?

Um diese Fragen zu beantworten, zerlegt man die planaren Strukturen in Teile mit höherer Konnektivität. Für die asymptotische Enumeration interpretiert man die Zerlegung mit Hilfe von generierenden Funktionen und dann verwendet man Singularitätenanalyse. Für die exakte Enumeration und zufällige Erzeugung verwendet man die sogenannte rekursive Methode. Für die typische Eigenschaften verwendet man die Probabilistische Methode bei asymptotischer Anzahl.

In meinem Vortrag zeige ich, wie man diese Methoden an einigen numerierten planaren Strukturen, z.B. outerplanaren Graphen, planaren Graphen und kubischen planaren Graphen, anwendet.

Amin Coja-Oghlan *(HU-Berlin)*
Central and local limit theorems for the giant component

Erdős and Rényi observed that in a random graph $G_{n,p}$ (or $G_{n,m}$) there occurs a *phase transition* as the average degree np (resp. $2m/n$) passes 1: if $np < 1-\epsilon$ for an arbitrarily small but fixed $\epsilon > 0$, then all connected components of $G_{n,p}$ have at most logarithmic size, while for $np > 1+\epsilon$ there is a "giant" component of linear size asymptotically almost surely as $n \to \infty$. Erdős and Rényi also computed the expected number of vertices/edges in the giant component. In this talk I present a novel approach to determining the actual *distribution* of the number of vertices/edges inside of the giant component. The techniques are purely probabilistic and include "Stein's method" as well as a bit of Fourier analysis. As a by-product, these techniques yield a new proof of Bender, Canfield, and McKay's forumla for the asymptotic number of connected graphs with a given number of vertices/edges.

This is joint work with Michael Behrisch and Mihyun Kang.

Dieter Rautenbach *(Bonn)*
Beyond Acyclic Colorings

Acyclic colorings of graphs have received a lot of attention in recent years and the probabilistic method has been applied to them quite successfully. The first notable probabilistic result about acyclic colorings is certainly Alon, McDiarmid and Reed's proof that every graph of maximum degree Δ can be acyclically (and properly) colored using $O(\Delta^{\frac{4}{3}})$ colors which implied a conjecture of Erdős. The deepest non-probabilistic result about acyclic colorings is probably Borodin's proof that every planar graph has an acyclic 5-coloring. Extending acyclicity Borodin conjectured in 1976 that every planar graph has a 5-coloring such that the union of every k color classes with $1 \leq k \leq 4$ induces a k-degenerate graph. We present results related to this conjecture.p

Mathias Schacht *(HU Berlin)*
On the bandwidth conjecture of Bollobás and Komlós

The study of sufficient degree conditions, on a given graph G, which imply that G contains a particular spanning subgraph H is one of the central areas in graph theory. A well known example of such a result is Dirac's theorem. It asserts that any graph G on n vertices with minimum degree at least $n/2$ contains a spanning, so called Hamiltonian, cycle.

In my talk we discuss related results for 3-chromatic graphs H of bounded maximum degree and small bandwidth. In particular we show that: *For every Δ and $\gamma > 0$ there exist a constant $\beta > 0$ such that for sufficiently large n the following holds. If G is an n-vertex graph with minimum degree $\delta(G) \geq (2/3 + \gamma)n$, then it contains a spanning copy of every 3-chromatic n-vertex graph H with maximum degree $\Delta(H) \leq \Delta$ and bandwidth $\mathrm{bw}(H)$ at most βn, where $\mathrm{bw}(H) = \min_\sigma \max_{uv \in E(H)} |\sigma(u) - \sigma(v)|$ with the minimum ranging over all bijections from $V(H)$ to $[n]$.* This settles a conjecture of Bollobás and Komlós for the special case of 3-chromatic graphs H. It is known that the minimum degree condition on G is asymptotically best possible.

The proof is based on Szemerédi's regularity lemma and the so called blow-up lemma.

This is joint work with Julia Böttcher and Anusch Taraz from TU München.