Minisymposium 25

Inverse Probleme und Inkorrektheits-Phänomene

Leiter des Symposiums:

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Inverse problems and ill-posedness phenomena as their intrinsic properties play a central role in several areas of mathematical research. Among others, they appear in connection with the detemination of system parameters from input-output measurements, and the reconstruction of not directly observable physical quantities from indirect measurements. The aim of the symposium is to bring together analytical, numerical and stochastical aspects of inverse problems theory. New theoretical results, interesting applications and numerical case studies will be presented.

Dienstag, 19. September

Seminarraum 17, AVZ I, Endenicher Allee 11-13

15:00 – 15:25 **Hans-Jürgen Reinhardt** *(Siegen)*
Approximate Solutions to Inverse Problems for Elliptic Equations

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15:30 – 15:55 **Sungwhan Kim** *(Daejeon-Korea)*
Inversion of Gravitational Potential Field Data: Isolated Interpretation Model

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16:00 – 16:25 **Thorsten Hohage** *(Göttingen)*
Convergence rate analysis of regularized Newton methods with random noise

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16:30 – 16:55 **Sybille Handrock-Meyer** *(Chemnitz)*
An inverse problem for the Grad-Schafranov equation

17:00 – 17:25 **Bernd Büchler** *(Kaiserslautern)*
The Error Localizing Property of Absolutely a-Compatible Operators

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17:30 – 17:55 **Bernd Hofmann** *(Chemnitz)*
Some new results on approximate source conditions

18:00 – 18:25 **Torsten Hein** *(Chemnitz)*
The potential of descriptive multiparameter regularization approaches

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18:30 – 18:55 **Horst Heck** *(Darmstadt)*
Stability Estimates for the Inverse Conductivity Problem with Partial Cauchy Data

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Vortragsauszüge

Hans-Jürgen Reinhardt *(Siegen)*

Approximate Solutions to Inverse Problems for Elliptic Equations

In this contribution we study Cauchy problems for 2-d. elliptic partial differential equations. These consist in determining a function – and its normal derivative – on one side of a rectangular domain from Cauchy data on the opposite side. With Cauchy data we mean the function itself and its normal derivative. On the other parts of the boundary Dirichlet or Neumann data are given. This type of problems is known to be illposed. Hadamard gave a classical example in 1923 demonstrating its illposedness. It should be noted that such Cauchy problems are conditionally well-posed which means that under certain restrictions on the data the problems are well-posed. Approximate solutions can be obtained by a semidiscretization of the rectangular domain which leads to a coupled system of boundary value problems for ordinary differential equations. The system can be decoupled by solving an eigenvalue problem in advance.
We study its stability (with respect to perturbations in the data) as well as the error behaviour in terms of the discretization parameter (for the semidiscretization).

References:

Charton, M., Reinhardt, H.-J.: Approximation of Cauchy problems for elliptic equations using the method of lines, WSEAS Transactions on Math., 4/1 (2005), 64-69.

Sungwhan Kim *(Daejeon-Korea)*

Inversion of Gravitational Potential Field Data: Isolated Interpretation Model

Gravity data inversion is an important technique for understanding the Earth's interior through its surface gravitational potential and has the wide variety of applications in exploration geophysics, mainly in finding deposits of ores, estimating the shape and depth extent of base of salt diapirs containing oil or gas, and determining the geological basement structure. Most of significant inverse problems in gravitational potential fields use one of three interpretation models : ore type, structural type or complicated distribution of sources (Mudretsova and Veselov ed., Gravirazvedka. Nedra. Moskva. (Gravimetry. Nedra, Moscow), p.228-235). We give our attention to the first one, ore type. Explorations of ores, deposits of oil and gas, and cavities through gravitational potential can be reduced to the ore category to determine the geometrical shape of isolated bodies with constant physical parameters. This presentation is devoted to mathematical analysis of gravity data inversion. We have a great interest in finding a prior information on a gravity source and understanding how much the information makes the gravity inversion problem stable. We also propose a quite simple reconstruction algorithm of a gravity source.

Thorsten Hohage *(Göttingen)*

Convergence rate analysis of regularized Newton methods with random noise

We consider the problem to estimate a quantity a in a separable Hilbert space given measurements of a function u related to a by $F(a) = u$ with a nonlinear operator F. The measurements are perturbed by random noise. In typical applications a is an unknown coefficient in a partial differential equation, and u is (part of) the solution to the differential equation.

We show that for regularized Newton methods the same rates of convergence can be achieved as for the underlying linear regularization method if the smoothness of the solution is known. For unknown smoothness we show that Lepskij's method yields a rate of convergence which differs from the optimal rate only by a logarithmic factor. The theoretical results are illustrated by numerical experiments.

Sybille Handrock-Meyer *(Chemnitz)*

An inverse problem for the Grad-Schafranov equation

This talk presents research which obtained as a joint work with A.S. Demidov (Moscow). We consider the Grad-Schafranov equation

$$\Delta u = a\,u + b \geq 0 \qquad \text{in} \qquad \Omega,$$

with homogeneous boundary conditions, where a and b are real constants. An inverse problem consists in the following: Under which conditions one can identify the constants a and b simultaneously from knowledge of the outward normal derivative. Some results in the cases $n = 2$ and $n = 3$ are discussed

Bernd Büchler *(Kaiserslautern)*
The Error Localizing Property of Absolutely a-Compatible Operators

On certain function spaces X compactly disturbed multiplication operators $T = \Lambda_a - K : X \to X$ usually lead to ill-posed inverse problems (T, X, X), if the multiplier function a has zeros on its domain of definition. In this context, we present a classification of compact perturbations K in dependence on the multiplier functions a, such that the corresponding ill-posed problems (T, X, X) behave like the reduced ill-posed problems of the form (Λ_a, X, X) with respect to a suitably chosen regularization method. Compact perturbations having the above mentioned property are called absolutely a-compatible – they lead to an error localizing phenomenon, that occurs in the framework of regularizing (T, X, X). We give examples and discuss classes of absolutely a-compatible operators in case of X being the Banach space of continuous functions on a compact interval and in case of X being the Hilbert space of square-integrable functions on a compact interval. Moreover, we explain for a special class of absolutely a-compatible operators the connection with the Lavrentiev resolvent condition. One will see, that dealing with ill-posed inverse problems (T, X, X) there is a strong interaction between multiplication operators, classes of absolutely a-compatible operators and suitably chosen regularization methods.

Bernd Hofmann *(Chemnitz)*
Some new results on approximate source conditions

This talk presents research which was in various combinations partly done in collaboration with Dana Düvelmeyer (TU Chemnitz), Peter Mathé (WIAS Berlin) and Masahiro Yamamoto (Univ. Tokyo). There are given some new ideas and results for finding convergence rates in regularization for ill-posed linear inverse problems with compact and non-compact forward operators based on the consideration of approximate source conditions. In this context, we exploit distance functions measuring the violation of a source condition that works as a benchmark. Under specific range inclusions the decay rate of distance functions is verified explicitly. Applications to non-compact multiplication operators are given. An important new result is that we can show for compact operators a one-to-one correspondence between the maximal power type decay rates for the distance functions and maximal exponents of Hölder rates in Tikhonov regularization linked by the specific singular value expansion of the solution element. Some numerical studies on simple integration illustrate the compact operator case and the specific situation of discretized problems.

Torsten Hein *(Chemnitz)*
The potential of descriptive multiparameter regularization approaches

Tikhonov-Phillips regularization is probably one of the most popular and best-understood regularization methods. Besides the regularization parameter which can be chosen for example by the discrepancy principle the choice of the penalty functional seems to be very crucial. In particular, if the solution which has to be determined has inhomogeneous properties or the noise-level is known only partially there probably does not exist an 'optimal' penalty term. We can overcome such problems with multi-parameter regularization. Instead a single regularization parameter and penalty we introduce a vector of regularization parameters with corresponding penalty functionals. The applications are various. So we can combine (partial) Tikhonov regularization with (partial) descriptive regularization approaches if we know a priori information about our solution. We present a discrepancy-like parameter choice based on Lagrangian techniques. An algorithm for solving the corresponding problem is proposed and illustrated by a numerical example.

Horst Heck *(Darmstadt)*
Stability Estimates for the Inverse Conductivity Problem with Partial Cauchy Data

We consider the parameter identification problem for the conductivity equation as well as for the Schrödinger equation using partial Cauchy data. We derive stability estimates for the local Dirichlet-to-Neumann maps associated with these inverse problems.