**Second exercise sheet "Algebra II" winter term 2024/5.**

**Problem 1** (5 points)**.** *Let $A$ be a UFD (i. e., a factorial domain) and $K$ its field of quotients. Show that $A$ is integrally closed in $A$.*

**Problem 2** (5 points)**.** *Let $L/K$ be a finite field extension, $V$ a finite-dimensional $L$-vector space and $A$ an endomorphism of $V$. Show that*

$$\det_K(A) = N_{L/K}\det_L(A),$$

*where $\det_K(A)$ and $\det_L(A)$ are the determinants of $A$ as an endomorphism of the $K$- or $L$-vector space $V$.*

**Problem 3** (5 points)**.** *Let $\overline{K}$ be algebraically closed, $K \subseteq \overline{K}$ a subfield over which $\overline{K}$ is algebraic and $L/K$ a finite separable field extension. Let $(\sigma_i)_{i=1}^d$, where $d = [L : K]$, be the $K$-linear embeddings $L \xrightarrow{\sigma_i} \overline{K}$. Show that we have an isomorphism of $\overline{K}$-vector spaces*

$$\overline{K} \otimes_K L \to \overline{K}^d$$
$$\kappa \otimes \lambda \to \big(\kappa\sigma_i(\lambda)\big)_{i=1}^d.$$

**Problem 4** (5 points)**.** *In the situation of the previous problem and for arbitrary $\ell \in L$, show that we have equalities in $\overline{K}$*

$$\mathrm{Tr}_{L/K}(\ell) = \sum_{i=1}^d \sigma_i(\ell)$$
$$\mathrm{N}_{L/K}(\ell) = \prod_{i=1}^d \sigma_i(\ell).$$

Solutions should be submitted to the tutor by e-mail before Friday October 25 24:00.