Problem sheet 3 Rigid analytic geometry Winter term 2024/25

Problem 1. *[6 points] Let X by any set.*

- *Let $(F_i)_{i\in I}$ be a family of subsets of X, with $F_i = \bigcup_{j\in J_i} F_{i,j}$, where J_i is finite. We assume $\bigcap_{i\in G} F_i \neq \emptyset$ for every finite subset $G \subseteq I$. Then there is a map ι defined on I such that $\iota(i) \in F_i$ for all $i \in I$ and such that the intersection of every finit subfamily of $\{F_{i,\iota(i)} \mid i \in I\}$ is non-empty.*
- *Let $(G_i)_{i\in I}$ be a family of subsets of X, with $G_i = \bigcap_{j\in J_i} G_{i,j}$, where J_i is finite. We assume that no finite subfamily of this family covers X. Then there is a map ι defined on I such that $\iota(i) \in F_i$ for all $i \in I$ and such that no finite subfamily of $\{G_{i,\iota(i)} \mid i \in I\}$ covers X.*

Problem 2 (11 points)**.** *Let X be a G_+-topological space and $\mathfrak{B}$ a G_+ topology base for X. Show that we have a bijection*

$$X^* \cong X^*_{\mathfrak{B}}$$
$$\xi \to \upsilon = \xi \cap \mathfrak{B}$$
$$\xi = \{U \in \mathfrak{O}_X \mid \mathfrak{B}_U \cap \upsilon \neq \emptyset\} \leftarrow \upsilon\}.$$

Problem 3 (4 points)**.** *Under the assumptions of the previous problem, show that $\{\Omega^* \mid \Omega \in \mathfrak{B}\}$ is a topology base for X^*.*

One of the 21 points for this sheet is a bonus point which does not count in the calculation of the 50%-limit for passing the exercises.

Solutions should be e-mailed to my institute e-mail address (my second name (franke) at math dot uni hyphen bonn dot de) before Monday November 11.