

Correction to:

Categories and orbispaces

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by Stefan Schwede

Markus Land has pointed out a false claim of mine regarding the essential image of the nerve functor $N : \mathbf{cat} \rightarrow \mathbf{sset}$ on the top of page 3175. While it is correct that the nerve of every small category is a 2-coskeletal simplicial set, the converse is not true in general. For example, the 2-coskeleton of the simplicial set $\Delta[2, 1]$, the inner horn of the simplicial 2-simplex, is 2-coskeletal (by design), but not the nerve of a small category. The false claim about the essential image of the nerve functor has no further consequences for the results of the paper.

Markus Land also pointed out that, moreover, the asserted characterization in Proposition 2.2.3 (iv) of Illusie's paper [18] is incorrect. The claim is that a simplicial set Y is isomorphic to the nerve of a small category if and only if Y is 2-coskeletal and in addition the square

$$\begin{array}{ccc} Y_2 & \xrightarrow{Y(d_0)} & Y_1 \\ Y(d_2) \downarrow & & \downarrow Y(d_1) \\ Y_1 & \xrightarrow{Y(d_0)} & Y_0 \end{array}$$

is a pullback. The answer by Mychael Smith to the mathoverflow question

<https://mathoverflow.net/questions/107951/>

entitled 'Which 2-coskeletal simplicial sets is the nerve of a category?' provides a counterexample to this claim.